

Critical current oscillation by magnetic field in semiconductor nanowire Josephson junction

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(Dated: August 4, 2014)

We study theoretically the critical current in semiconductor nanowire Josephson junction with strong spin-orbit interaction. The critical current oscillates by an external magnetic field. We reveal that the oscillation of critical current depends on the orientation of magnetic field in the presence of spin-orbit interaction. We perform a numerical simulation for the nanowire by using a tight-binding model. The Andreev levels are calculated as a function of phase difference φ between two superconductors. The DC Josephson current is evaluated from the Andreev levels in the case of short junctions. The spin-orbit interaction induces the effective magnetic field. When the external field is parallel with the effective one, the critical current oscillates accompanying the $0-\pi$ like transition. The period of oscillation is longer as the angle between the external and effective fields is larger.

PACS numbers:

I. INTRODUCTION

The spin-orbit (SO) interaction has attracted a lot of interest. In narrow-gap semiconductors, such as InAs and InSb, the strong SO interaction has been reported and many phenomena based on the SO interaction are investigated intensively, e.g., spin Hall effect¹. The SO interaction has a great advantage also for the application to the spintronic devices and to quantum information processing. InAs and InSb nanowires are interesting platform for the application and studied in recent experiments, e.g., the electrical manipulation of single electron spins in quantum dots fabricated on the nanowires². Nanowire-superconductor hybrid systems have been also examined for the search of Majorana fermions induced by the SO interaction and the Zeeman effect³.

In Josephson junctions, the supercurrent flows when the phase difference φ between two superconductors is present. In this paper, we investigate theoretically the Josephson junction of semiconductor nanowires with strong SO interaction. The supercurrent in semiconductor nanowires has been reported by experiment groups^{4,5}. The Josephson effect with SO interaction has been studied theoretically for some materials, e.g., magnetic normal metals⁶, where the combination of SO interaction and exchange interaction results in an unconventional current-phase relation, $I(\varphi) = I_0 \sin(\varphi - \varphi_0)$. The phase shift φ_0 deviates the ground state of junction from $\varphi = 0$ or π , which is so-called φ_0 -state. The anomalous supercurrent is obtained at $\varphi = 0$. In previous studies, we have pointed out that the anomalous effect is attributed to the spin-dependent channel mixing due to the SO interaction^{7,8}. In the present study, we focus on the critical current oscillation when an external magnetic field is applied. The DC Josephson current is evaluated from the Andreev levels in the case of short junction. We examine a numerical calculation using a tight-binding model for the nanowire. In this model, a particular form of SO interac-

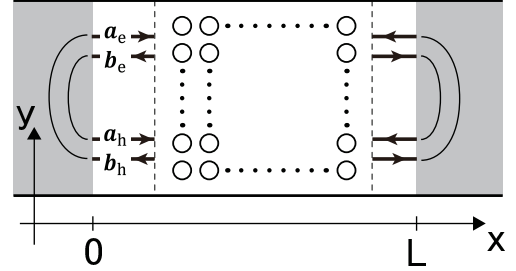


FIG. 1: Model for a semiconductor nanowire Josephson junction. A tight-binding model is applied for the normal region with hard-wall potentials for the nanowire. The nanowire is represented by a quasi-one dimensional system along the x direction. The normal region is $0 < x < L$ and the superconducting region induced by the proximity effect is $x < 0$, $x > L$.

tion can be considered. When the angle between the external field and an effective magnetic field due to the SO interaction is smaller, the oscillation period of critical current is shorter.

II. MODEL

The nanowire along the x direction is connected to two superconductors (Fig. 1). At $x < 0$ and $x > L$, the superconducting pair potential is induced into the nanowire by the proximity effect. We assume that the pair potential is $\Delta(x) = \Delta_0 e^{i\varphi/2}$ at $x < 0$ and $\Delta(x) = \Delta_0 e^{-i\varphi/2}$ at $x > L$, where φ is the phase difference between the two superconductors. In the normal region at $0 < x < L$, $\Delta(x) = 0$. When a magnetic field is applied to the junction, the Zeeman effect is taken into account in the nanowire. The magnetic field is not too large to break the superconductivity and screened in the superconducting regions. The Hamiltonian is given by $H = H_0 + H_{SO} + H_Z$ with $H_0 = \mathbf{p}^2/2m + V_{\text{conf}} + V_{\text{imp}}$, the

Rashba interaction $H_{SO} = (\alpha/\hbar)(p_y\sigma_x - p_x\sigma_y)$, and the Zeeman term $H_Z = g\mu_B \mathbf{B} \cdot \hat{\sigma}/2$, using effective mass m , g -factor g (< 0 for InSb), Bohr magneton μ_B , and Pauli matrices $\hat{\sigma}$. We neglect the orbital magnetization effect in the nanowire. V_{conf} describes the confining potential for the nanowire. V_{imp} represents the impurity potentials. We consider short junction, where the spacing between two superconductors is much smaller than the coherent length in the normal region, $L \ll \xi$. There is no potential barrier at $x = 0, L$. The Zeeman energy $E_Z = |g\mu_B B|$ and the pair potential Δ_0 are much smaller than the Fermi energy E_F .

The Bogoliubov-de Gennes (BdG) equation is written as

$$\begin{pmatrix} H - E_F & \hat{\Delta} \\ \hat{\Delta}^\dagger & -(H^* - E_F) \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_h \end{pmatrix} = E \begin{pmatrix} \psi_e \\ \psi_h \end{pmatrix} \quad (1)$$

with $\hat{\Delta} = \Delta(x)\hat{g}$. $\psi_e = (\psi_{e+}, \psi_{e-})^T$ and $\psi_h = (\psi_{h+}, \psi_{h-})^T$ are the spinors for electron and hole, respectively. $\hat{g} = -i\hat{\sigma}_y$. The energy E is measured from the Fermi level E_F . The BdG equation determines the Andreev levels E_n ($|E_n| < \Delta_0$) as a function of φ .

The ground state energy of junction is given by $E_{\text{gs}}(\varphi) = -(1/2)\sum_n' E_n(\varphi)$, where the summation is taken over all the positive Andreev levels, $E_n(\varphi) > 0$. The contribution from continuous levels ($|E| > \Delta_0$) can be disregarded for the short junctions⁹. At zero temperature, the supercurrent is calculated as $I(\varphi) = (2e/\hbar)(dE_{\text{gs}}/d\varphi)$. The current is a periodic function for $-\pi \leq \varphi < \pi$. The maximum (or absolute value of minimum) of $I(\varphi)$ yields the critical current I_c .

The BdG equation in eq. (1) is expressed in terms of the scattering matrix⁹. The scattering matrix of electrons (holes) transport in the normal region is given by \hat{S}_e (\hat{S}_h). \hat{S}_e and \hat{S}_h are related to each other by $\hat{S}_e = \hat{S}_h^*$ for the short junctions. We denote $\hat{S}_e = \hat{S}$ and $\hat{S}_h = \hat{S}^*$. The Andreev reflection at $x = 0$ and L is described by the scattering matrix \hat{r}_{he} for the conversion from electron to hole and \hat{r}_{eh} for that from hole to electron. The normal reflection can be neglected. The matrix coefficients of \hat{r}_{he} and \hat{r}_{eh} , e.g., $\exp\{-i \arccos(E/\Delta_0) - i\varphi/2\}$ for \hat{r}_{he} at $x = 0$, are calculated from the boundary condition at $x = 0$ and L . The SO interaction does not affect the Andreev reflection coefficients. The Andreev levels, $E_n(\varphi)$, are obtained from the product of \hat{S} , \hat{r}_{he} , and \hat{r}_{eh} ,

$$\det(\hat{1} - \hat{r}_{\text{eh}}\hat{S}^*\hat{r}_{\text{he}}\hat{S}) = 0. \quad (2)$$

Equation (2) is equivalent to the BdG equation (1).

To calculate the scattering matrix \hat{S} , we adopt the tight-binding model which discretizes a two-dimensional space (xy plane). The edges of nanowire are represented by a hard-wall potential. The width of nanowire is $W = 12a$ with the lattice constant $a = 10\text{nm}$. The Fermi wavelength is fixed at $\lambda_F = 18a$, where the number of conduction channels is unity. The length of normal region is $L = 50a$. The on-site random potential by impurities is taken into account, the distribution of which potential is uniform. We set that the mean free path due to the impurity scattering is $l_{\text{mfp}}/L = 1$. The SO length is $l_{SO}/L = 0.2$ with $l_{SO} = k_\alpha^{-1} = \hbar^2/(m\alpha)$. The magnetic field is $\mathbf{B} = B\hat{e}_\theta$ with the angle θ from the x axis in the xy plane.

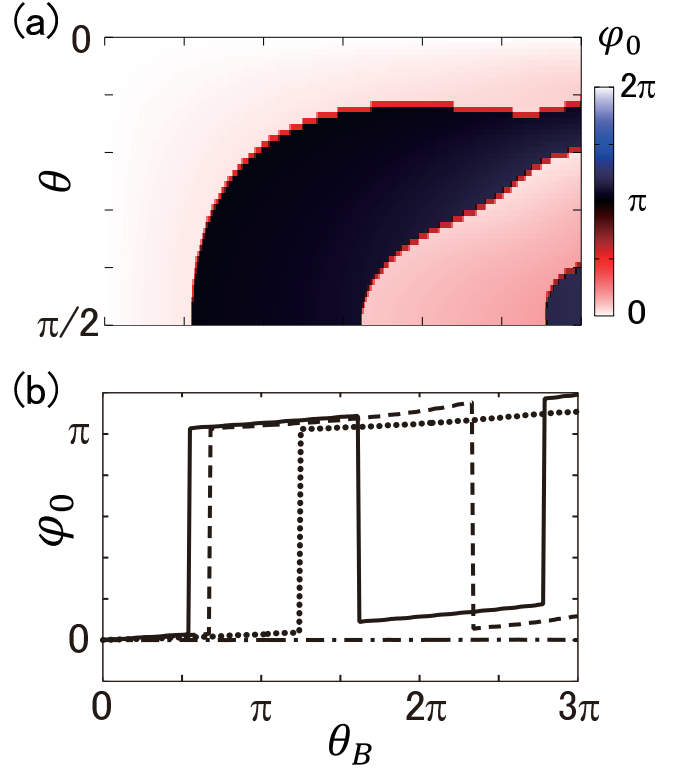


FIG. 2: Numerical results of phase difference φ_0 at the minimum of ground-state energy when the number of conduction channel is unity and $l_{\text{mfp}}/L = 1$. The SO interaction is $l_{SO}/L = 0.2$. The results are for a sample. (a) Grayscale plot of φ_0 in the plane of magnetic field $\theta_B = E_Z L/(\hbar v_F)$ and its direction θ . (b) Cross section of panel (a) at $\theta = 0.5\pi$ (solid), 0.3π (broken), 0.15π (dotted), and 0 (dotted broken lines).

III. RESULTS

We consider a sample for the nanowire. For the magnetic field, we introduce a parameter, $\theta_B = E_Z L/(\hbar v_F)$, which means an additional phase due to the Zeeman effect in the propagation of electron and hole. Here, v_F is the Fermi velocity in the absence of SO interaction.

Figure 2 shows the phase difference φ_0 at the minimum of E_{gs} when the magnetic field is increased and rotated. In the absence of SO interaction, φ_0 takes only 0 or π exactly and clear $0-\pi$ transition happens (see Ref.⁸). In the presence of SO interaction, φ_0 is deviated from 0 and π , where the anomalous Josephson current is obtained. When the magnetic field is in the y direction ($\theta = \pi/2$), the transition between $\varphi_0 \approx 0$ and $\varphi_0 \approx \pi$ takes place around $\theta_B = \pi/2, 3\pi/2, \dots$. The transition points are shifted gradually to large θ_B with decreasing of angle θ . At $\theta \approx 0$, the transition is not observed in Fig. 2.

The transition points in φ_0 corresponds to the position of cusps of critical current. Figure 3(a) exhibits the critical current when the magnetic field increases. The critical current I_c oscillates as a function of θ_B . The distance of cusps is longer when the direction of magnetic field is tilted from the y axis. For the parallel magnetic field to the nanowire ($\theta = 0$), I_c has

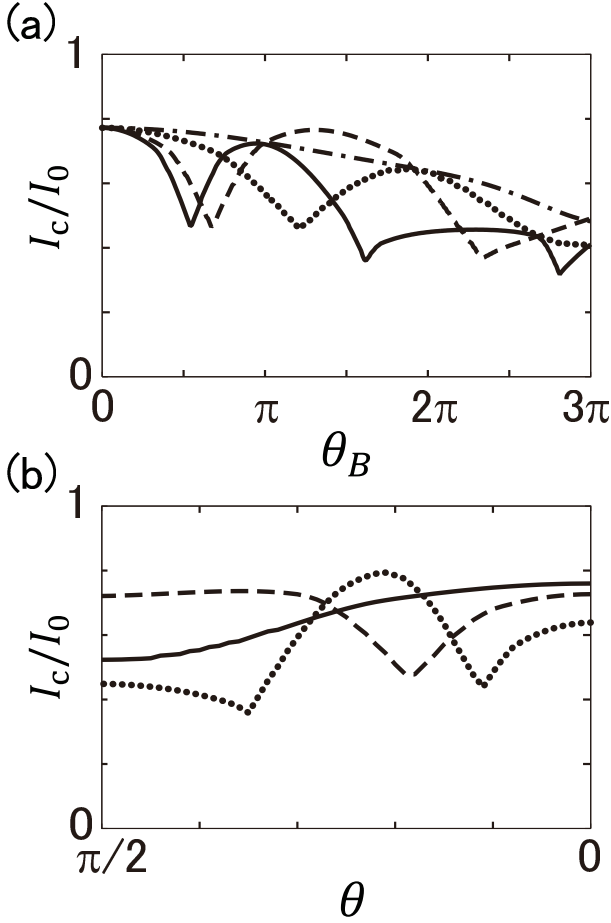


FIG. 3: Numerical results of critical current I_c when $N = 1$ and $I_{\text{mfp}}/L = 1$. The SO interaction is $I_{\text{SO}}/L = 0.2$. $I_0 \equiv e\Delta_0/\hbar$. The sample is the same as that in Fig. 2. (a) I_c as a function of magnetic field $\theta_B = E_Z L/(\hbar v_F)$ when $\theta = 0.5\pi$ (solid), 0.3π (broken), 0.15π (dotted), and 0 (dotted broken lines). (b) I_c as a function of magnetic field orientation θ when $\theta_B = \pi/2$ (solid), π (broken), and 2π (dotted lines).

no cusp in accordance with no transition. The critical current decreases with increase of θ_B although φ_0 is almost fixed at zero in Fig. 2(b). Figure 3(b) shows I_c when the magnetic field is rotated in the xy plane. The strength of magnetic field is fixed for each line. We find the transition by the angle θ in Fig. 2(a). The critical current also oscillates as a function of θ . For small magnetic field ($\theta_B < \pi/2$), I_c changes monotonically. The oscillation by θ is obtained for large magnetic field. If the state of junction at $\theta = \pi/2$ is $\varphi_0 \approx \pi$ (0), the critical current shows one cusp (two cusps) in Fig. 3(b). Therefore we can estimate the state at $\theta = \pi/2$ from the number of cusps.

IV. CONCLUSIONS AND DISCUSSION

We have studied the DC Josephson effect in the semiconductor nanowire with strong SO interaction. We have examined a numerical simulation using the tight-binding model in the case of short junction. The combination of SO interaction and Zeeman effect in the nanowire results in the anomalous Josephson effect. The critical current oscillates as a function of magnetic field. In the presence of SO interaction, the oscillation of critical current depends on the magnetic field orientation. The oscillation period is shorter when the magnetic field is perpendicular to the nanowire. For a parallel magnetic field to the nanowire, the transition and the cusp of critical current are not found.

In this numerical model, we have considered the Rashba interaction. In the quasi-one-dimensional nanowire, the effective magnetic field induced by the Rashba interaction is in the y direction. The anisotropy of critical current oscillation is understood intuitively by the spin precession in the propagation of electron and hole. When the external magnetic field is parallel to the effective field (the y direction), the spin quantization axis is fixed in that direction. The electron and hole receive the additional phase in the propagation. On the other hand, when the external field is in the x direction, the spin quantization axes for electron and hole are not parallel with each other since the effective fields for electron and hole are antiparallel. The spin of electron and hole forming the Andreev bound state is rotated, which rotation cancels out the phase θ_B due to the Zeeman splitting. As a result, the critical current oscillation disappears. In the case of general SO interaction, the effective field would be deviated from the y axis. By measuring the anisotropy of critical current oscillation, we can evaluate the direction of effective field due to the SO interaction.

Acknowledgments

We acknowledge financial support by the Motizuki Fund of Yukawa Memorial Foundation. We acknowledge fruitful discussions about experiments with Professor L. P. Kouwenhoven, A. Geresdi, V. Mourik, K. Zuo of Delft University of Technology. T.Y. is a JSPS Postdoctoral Fellow for Research Abroad.

References

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¹ Kato Y K, Myers R C, Gossard A C, and Awschalom D D 2004 *Science* **306** 1910

² Nadj-Perge S, Pribiag V S, van den Berg J W G, Zuo K, Plissard S R, Bakkers E P A M, Frolov S M, and Kouwenhoven L P 2012 *Phys. Rev. Lett.* **108** 166801

- ³ Mourik V, Zuo K, Frolov S M, Plissard S R, Bakkers E P A M, and Kouwenhoven L P 2012 *Science* **336** 1003
- ⁴ Doh Y-J, van Dam J A, Roest A L, Bakkers E P A M, Kouwenhoven L P, and Franceschi S De 2005 *Science* **309** 272
- ⁵ Kouwenhoven L P, Geresdi A, Mourik V, and Zuo K (private communications)
- ⁶ Buzdin A 2008 *Phys. Rev. Lett.* **101** 107005
- ⁷ Yokoyama T, Eto M, and Nazarov Yu V 2013 *J. Phys. Soc. Jpn.* **82** 054703
- ⁸ Yokoyama T, Eto M, and Nazarov Yu V 2014 *Phys. Rev. B* **89** 195407
- ⁹ Beenakker C W J 1991 *Phys. Rev. Lett* **67** 3836; 1992 *Phys. Rev. Lett* **68** 1442(E)